UNIVERSITY OF CAMBRIDGE

Solid State Devices

4B6

Lecture 2 – MOS capacitor (i)

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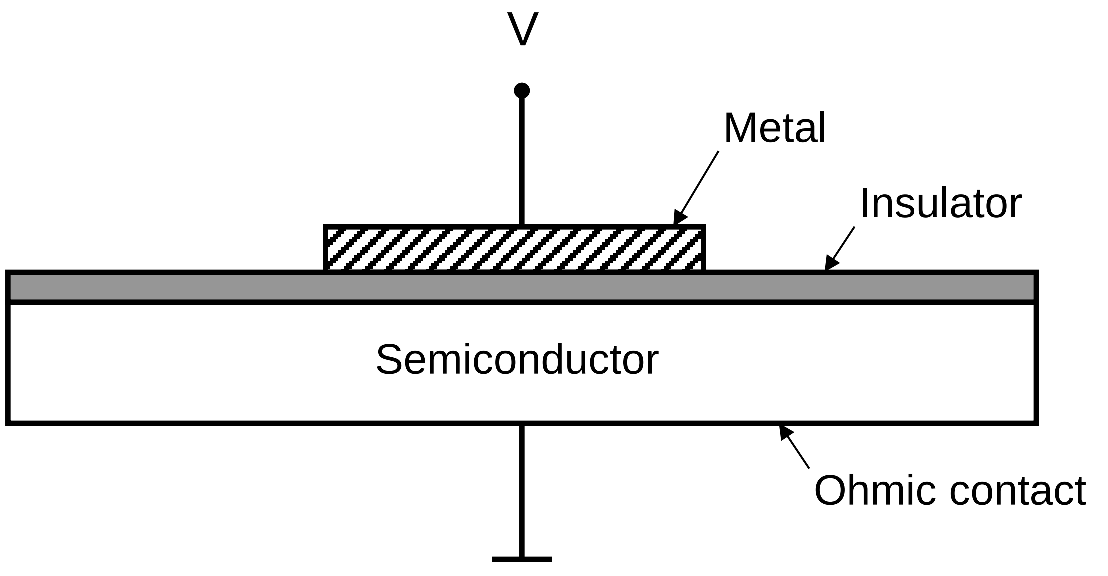
Lent 2016

Fundamentals

Metal-Insulator-Semiconductor (MIS) capacitor:



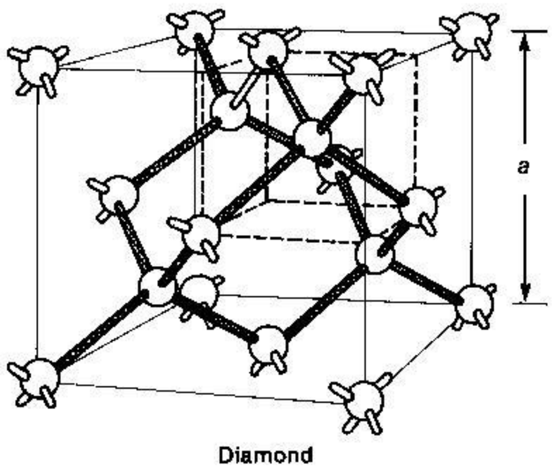
Insulator => Oxide   MIS capacitor => MOS capacitor

Objectives:

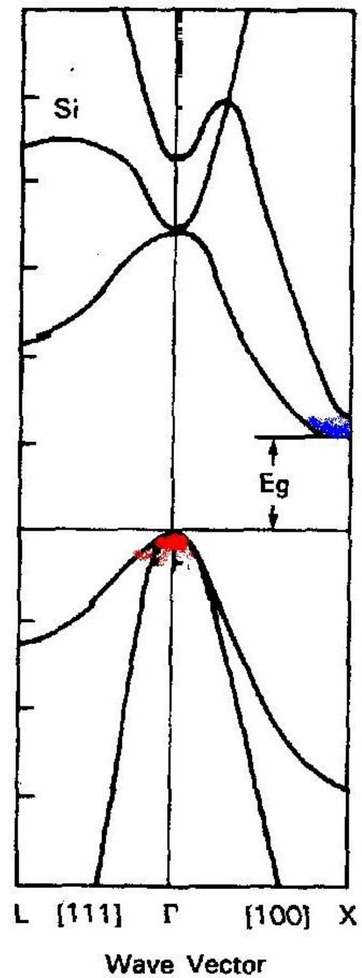
- To determine the relationship Q(V) between Charge Q and applied Voltage V;
- To determine the relationship C(V) between Capacitance C and applied Voltage V.

Fundamentals

Silicon crystalline and energy band structures



Tetrahedrally bonded, with high mechanical strength and chemical inertness.



Conduction band

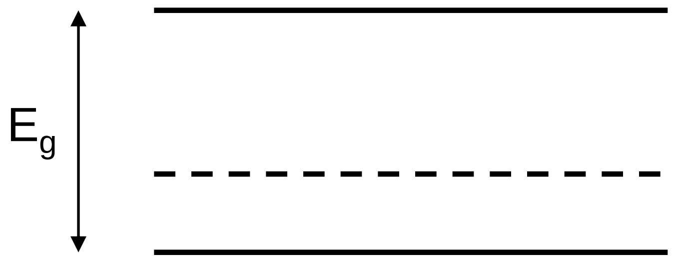
Valence band

Indirect band gap, which is not ideal for optoelectronics

Fundamentals

Semiconductor material:

Energy band



E_C Energy of conduction band minimum

E_F Fermi energy

E_V Energy of valence band maximum

Fermi distribution:

$$F(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

$$E_V + 3kT \leq E_F \leq E_C - 3kT$$

Non-degenerate semiconductor

$$n = N_C\, e^{-\frac{E_C - E_F}{kT}}$$

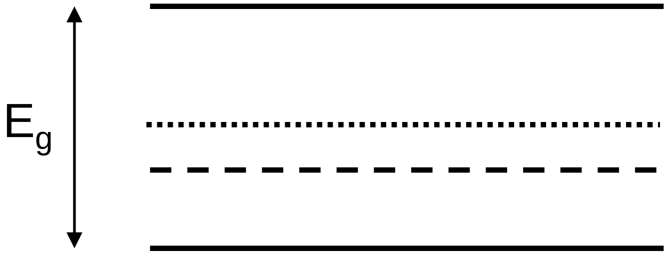
$$p = N_V\, e^{-\frac{E_F - E_V}{kT}}$$

where N_C (N_V) is the effective density of states in the conduction (valence) band.

Fundamentals

Semiconductor material:

Energy band



E_C Energy of conduction band minimum

E_i Intrinsic energy

E_F Fermi energy

E_V Energy of valence band maximum

Intrinsic semiconductor:

$n = p = n_i$

$E_F = E_i$

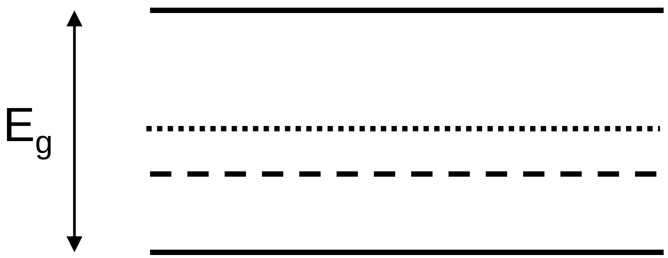
$$E_i = E_V + \frac{1}{2} E_g + \frac{1}{2} kT \ln\left(\frac{N_V}{N_C}\right)$$

$$n_i = \sqrt{np} = \sqrt{N_C N_V}\, e^{-\frac{E_g}{2kT}}$$

Fundamentals

Semiconductor material:

Energy band



E_C Energy of conduction band minimum

E_i Intrinsic energy

E_F Fermi energy

E_V Energy of valence band maximum

Alternatively:

n = p = n_i when **$E_F = E_i$**

$$n_i = N_C\, e^{-\frac{E_C - E_i}{kT}}$$

$$n_i = N_V\, e^{-\frac{E_i - E_V}{kT}}$$

$$n = n_i\, e^{\frac{E_F - E_i}{kT}}$$

$$p = n_i\, e^{\frac{E_i - E_F}{kT}}$$

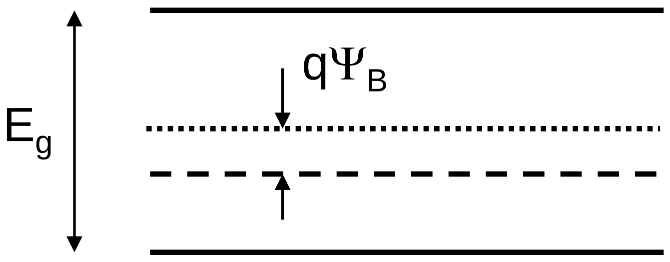
$$n_i = \sqrt{N_C N_V}\, e^{-\frac{E_g}{2kT}}$$

$$np = n_i^2$$

Fundamentals

Semiconductor material:

Energy band



E_C Energy of conduction band minimum

E_i Intrinsic energy

E_F Fermi energy

E_V Energy of valence band maximum

Valid always, equilibrium or not:

$$n = n_i \, e^{\frac{E_F - E_i}{kT}}$$

$$p = n_i \, e^{\frac{E_i - E_F}{kT}}$$

In equilibrium, $q\Psi_B = E_i - E_F$

$$n = n_i \, e^{-\frac{q\Psi_B}{kT}}$$

$$p = n_i \, e^{\frac{q\Psi_B}{kT}}$$

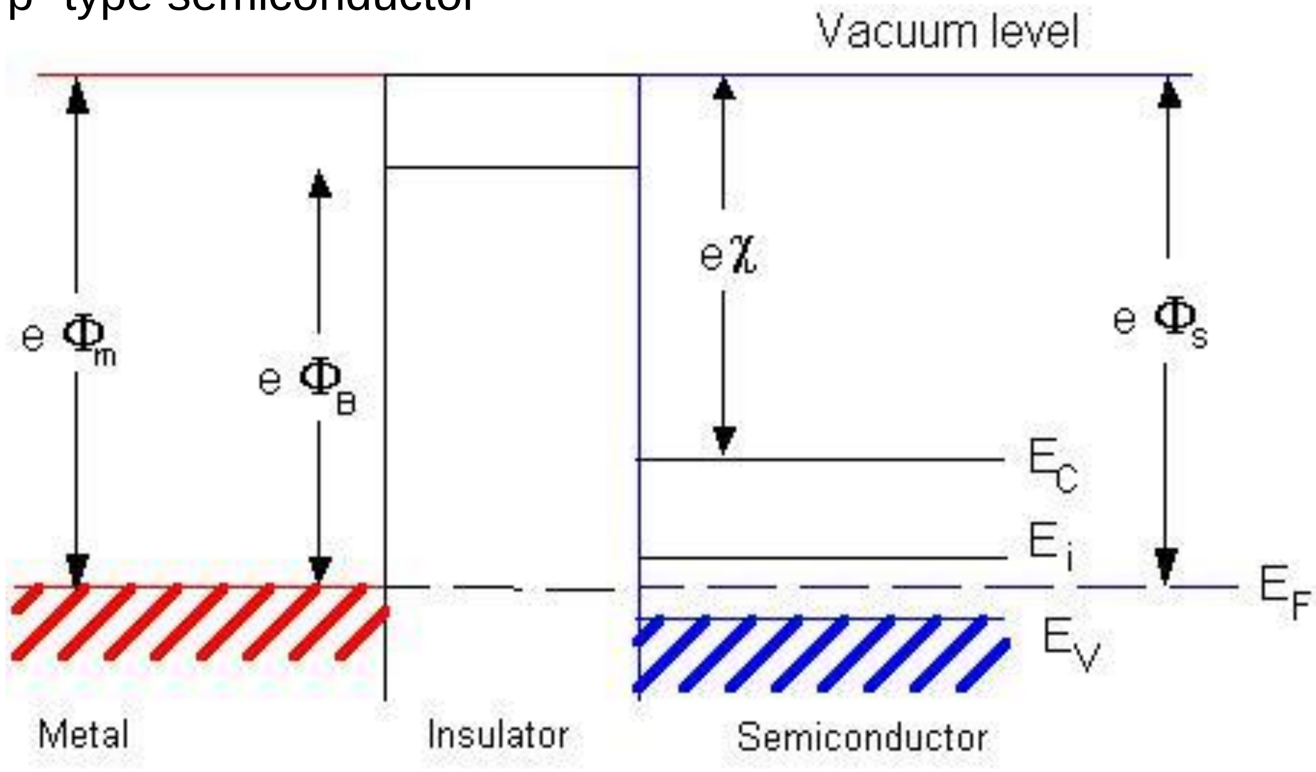
Ideal MOS capacitor

Assumptions:

- Equal work function for metal and semiconductor.
- Ideal insulator (oxide):
  - no trapped charge inside or at interfaces.
  - no carrier transport (infinite resistance).
- Spatially uniform materials:
  - flat band conditions / energy levels independent of position.

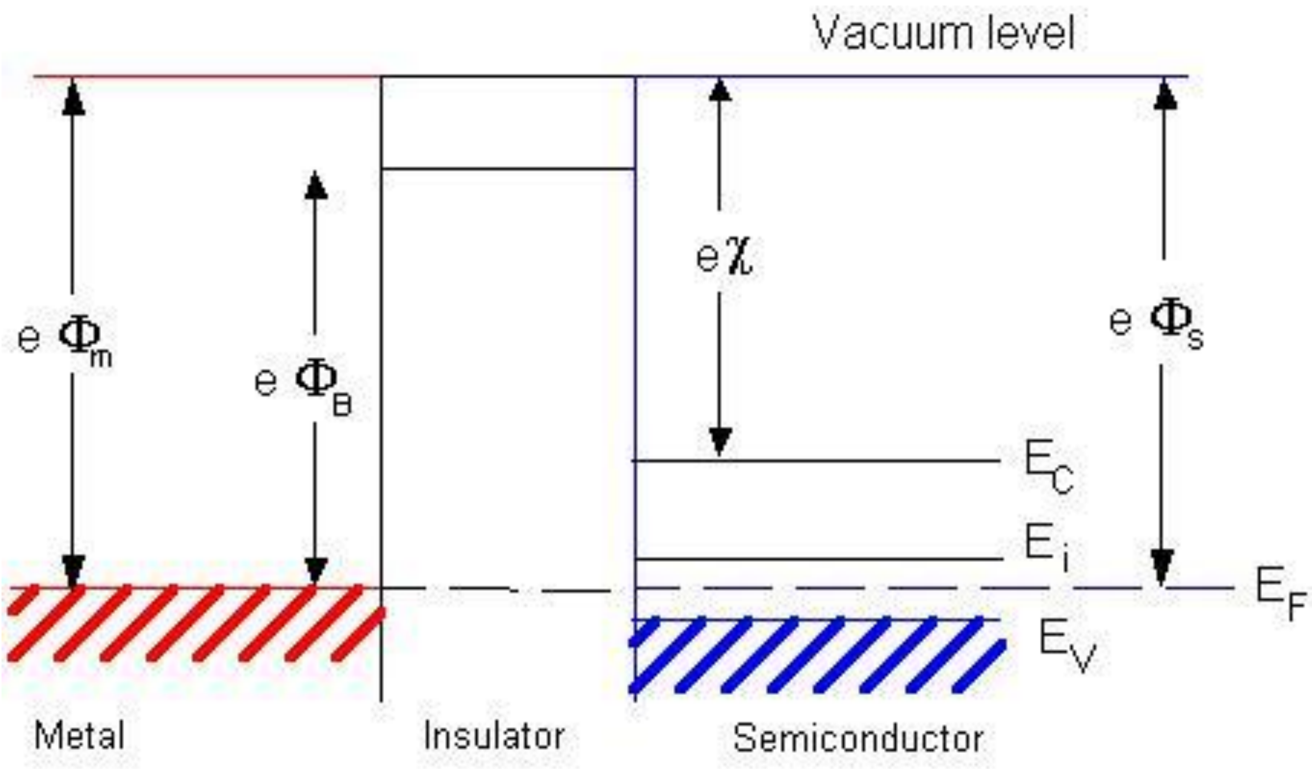
Ideal MOS capacitor

Example: p- type semiconductor



Vacuum level	energy of an electron immediately outside the solid.
Φ_m	work function for metal.
Φ_s	work function for semi.
χ	electron affinity for semi.
E_i	Fermi energy for intrinsic semi.

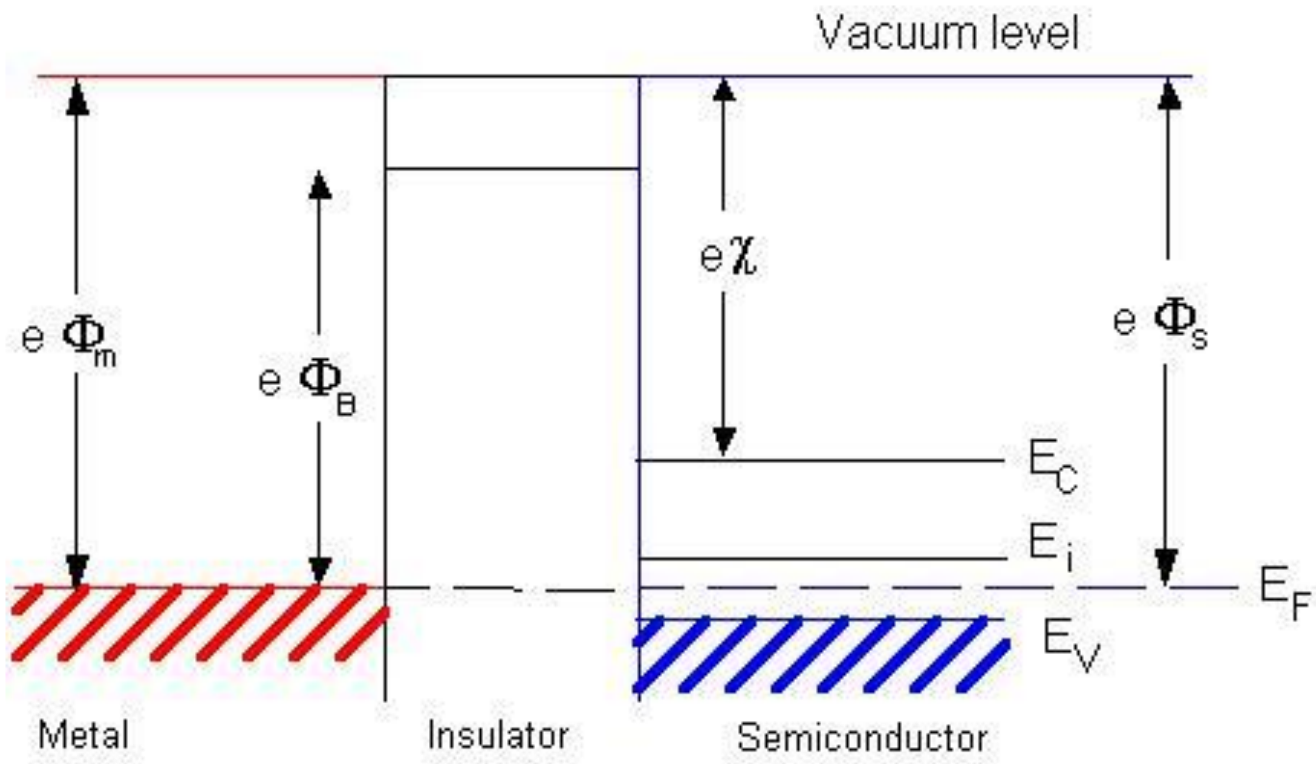
Ideal MOS capacitor



In equilibrium, potentials related by $e\Phi_s = e\chi + \frac{1}{2}E_g + e\Psi_B$

E_g	energy band gap between E_C and E_V,
$e\psi_B$	diff. between E_i and E_F.

Ideal MOS capacitor

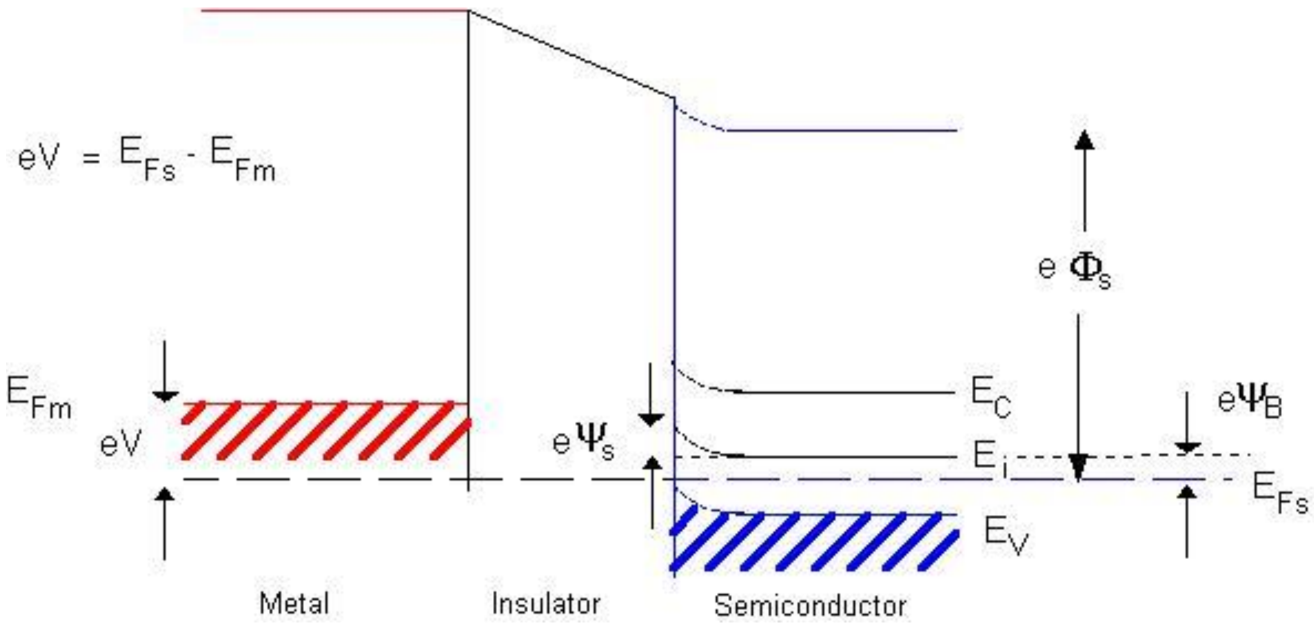


In flat band condition, the Fermi level is equal in metal and semiconductor, with no applied bias voltage.

Now apply a potential difference **V** between the metal and the semiconductor..................

Ideal MOS capacitor in accumulation

Negative gate voltage accumulates holes at the semiconductor-oxide interface.



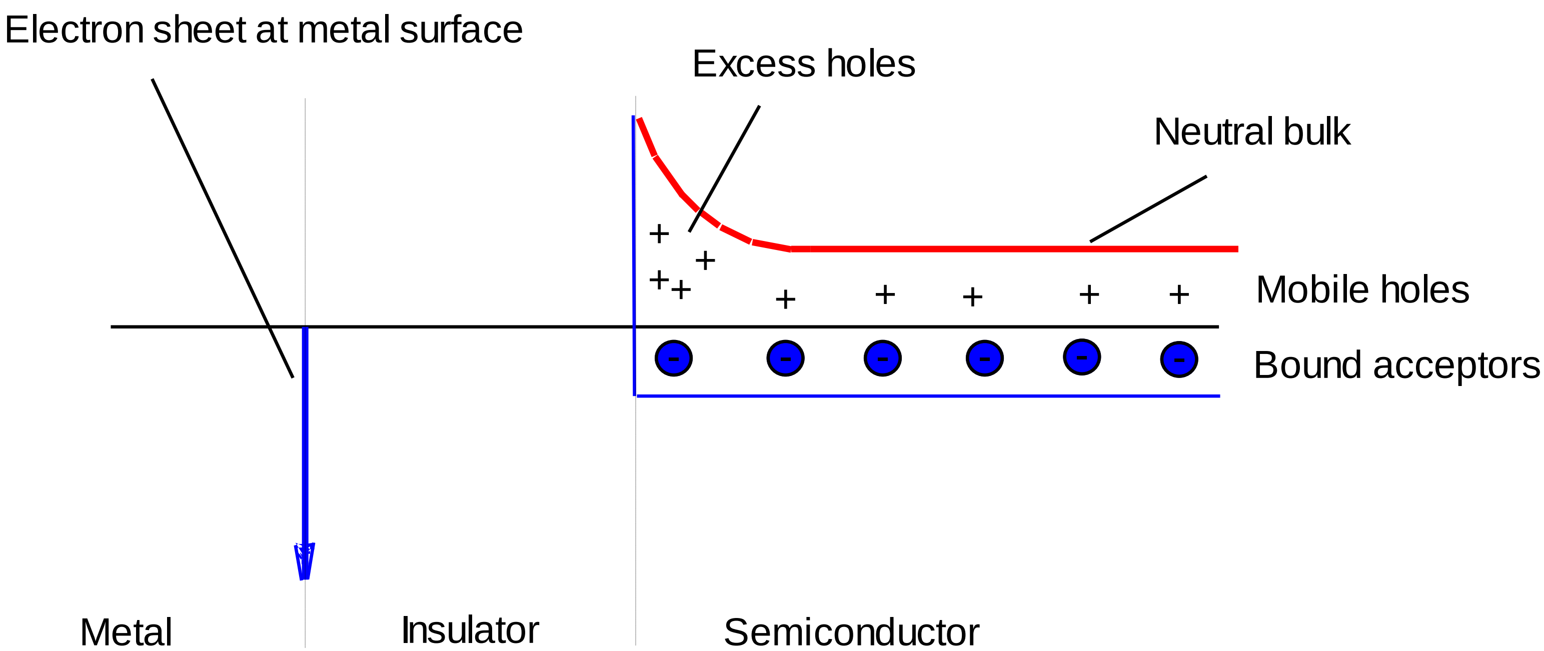
Neutrality maintained, excess electrons in metal = excess holes in semi.

Vacuum level is continuous and electric field exists in the insulator.

Away from interfaces, energy levels are displaced by **eV**.

Ideal MOS capacitor in accumulation

Charge distribution in the structure

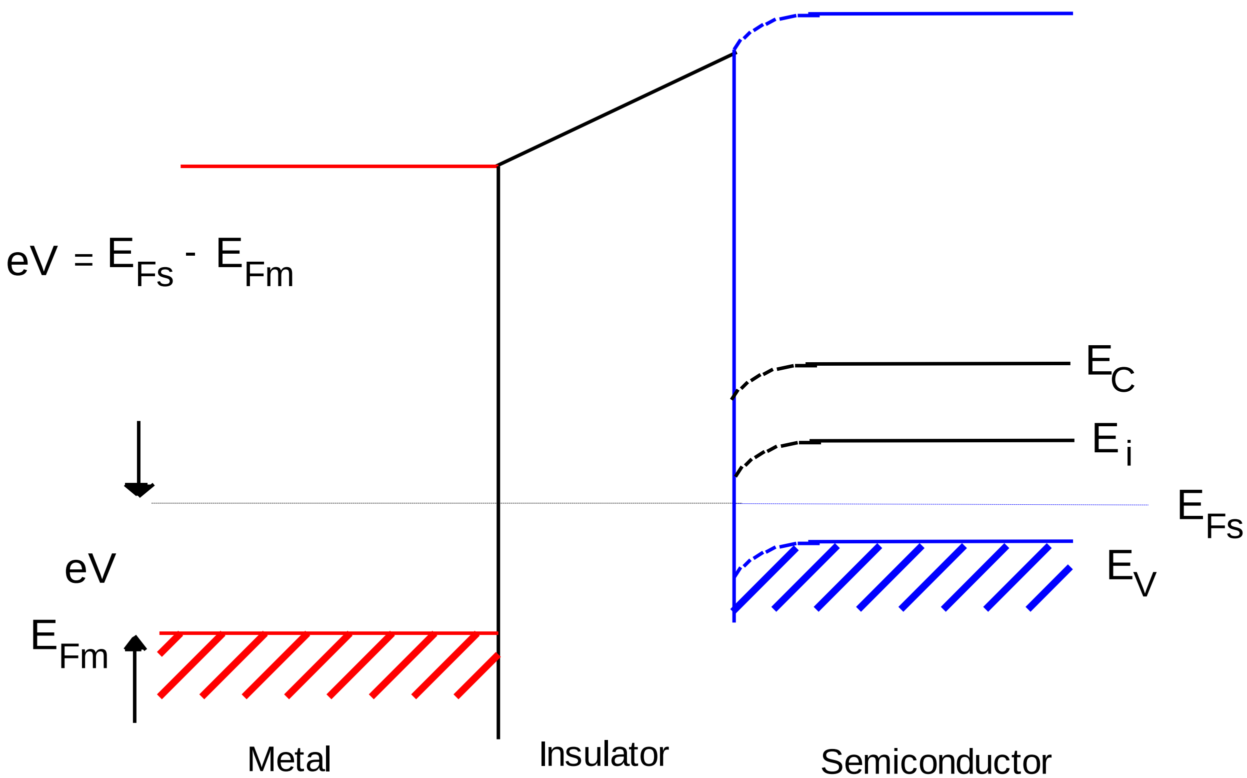


The hole density is given by:

$$p_p = n_i e^{\left[\frac{(E_i - E_F)}{kT}\right]}$$

Ideal MOS capacitor in depletion

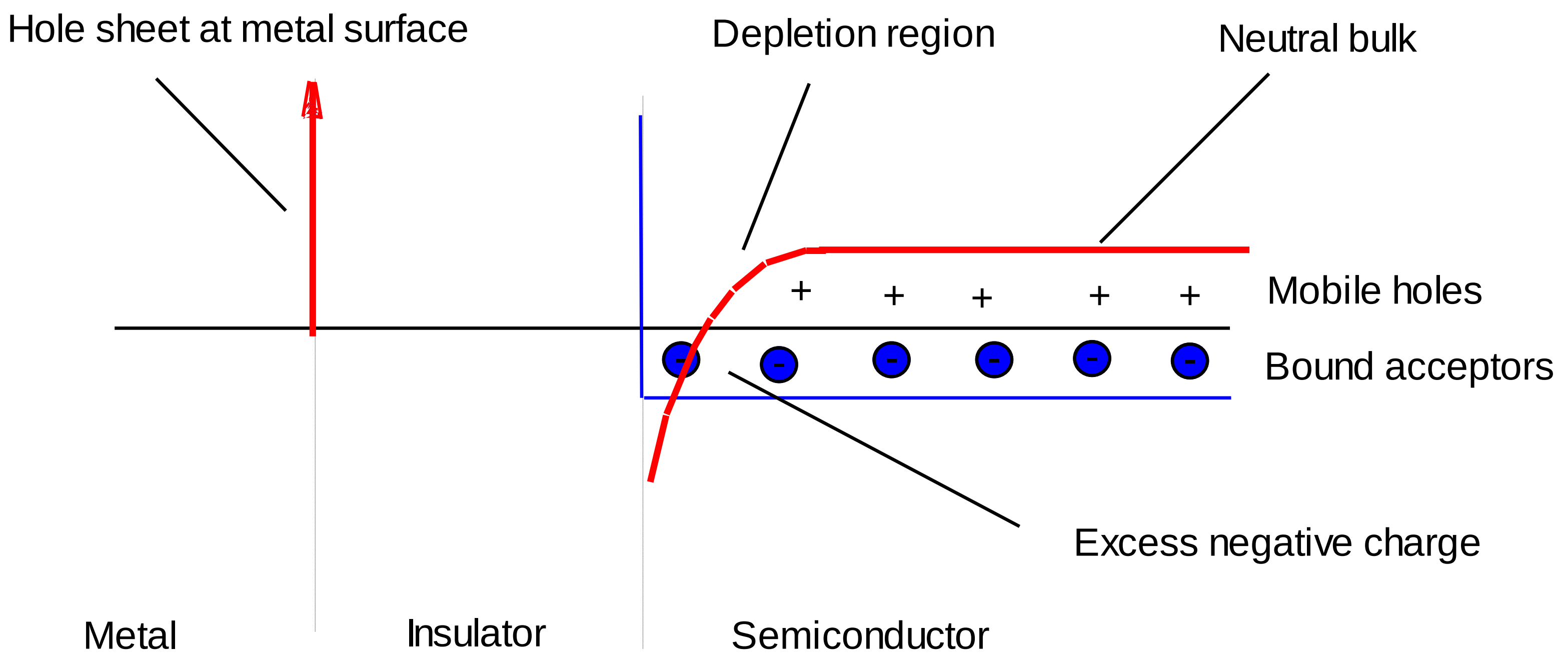
Small positive gate voltages,



Hole concentration in the semiconductor near to the interface is less than in the bulk.

Ideal MOS capacitor in depletion

Charge distribution in the structure



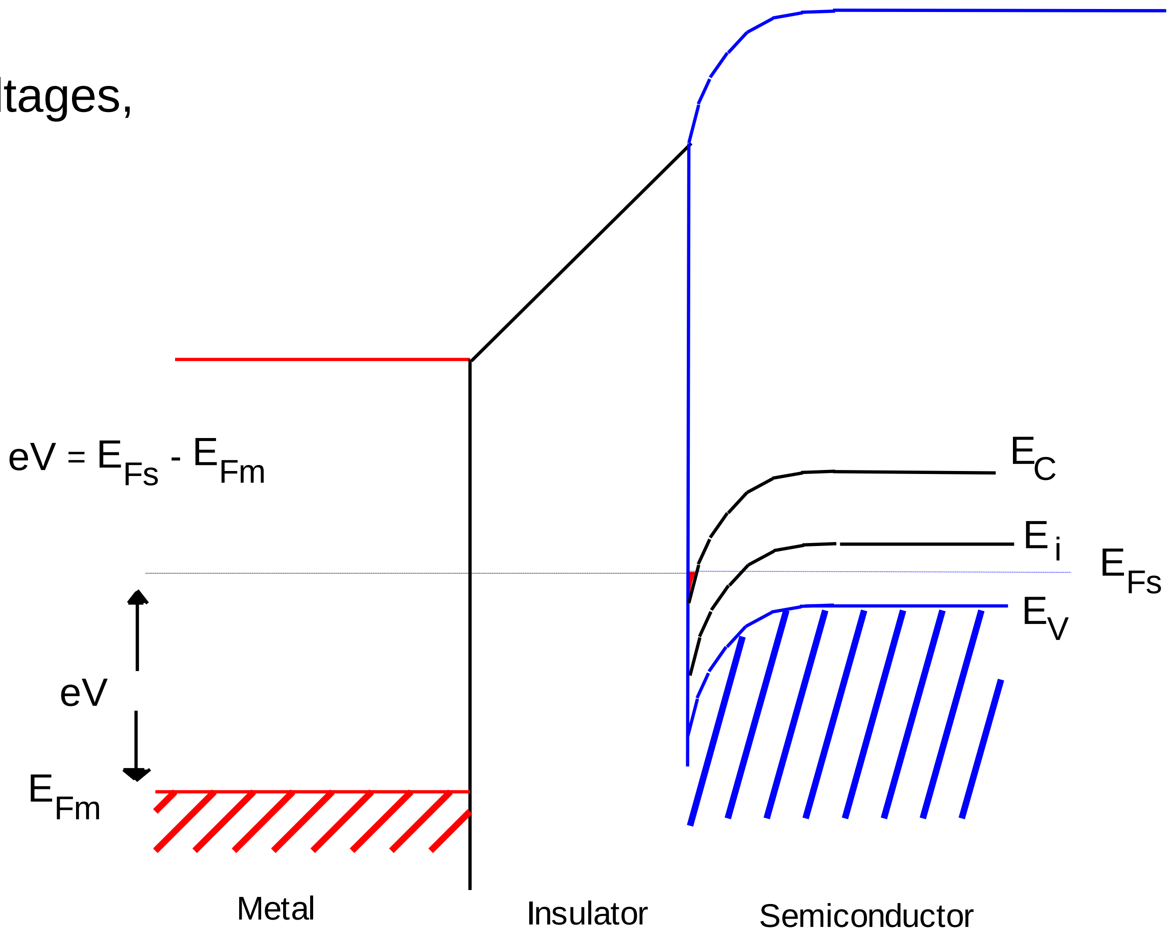
The hole density is given by:

$$p_p = n_i e^{\left[\frac{(E_i - E_F)}{kT}\right]}$$

Ideal MOS capacitor in inversion

Large positive gate voltages,

E_i line crosses **E_{Fs}** hence the electron concentration near to the oxide is greater than the hole concentration

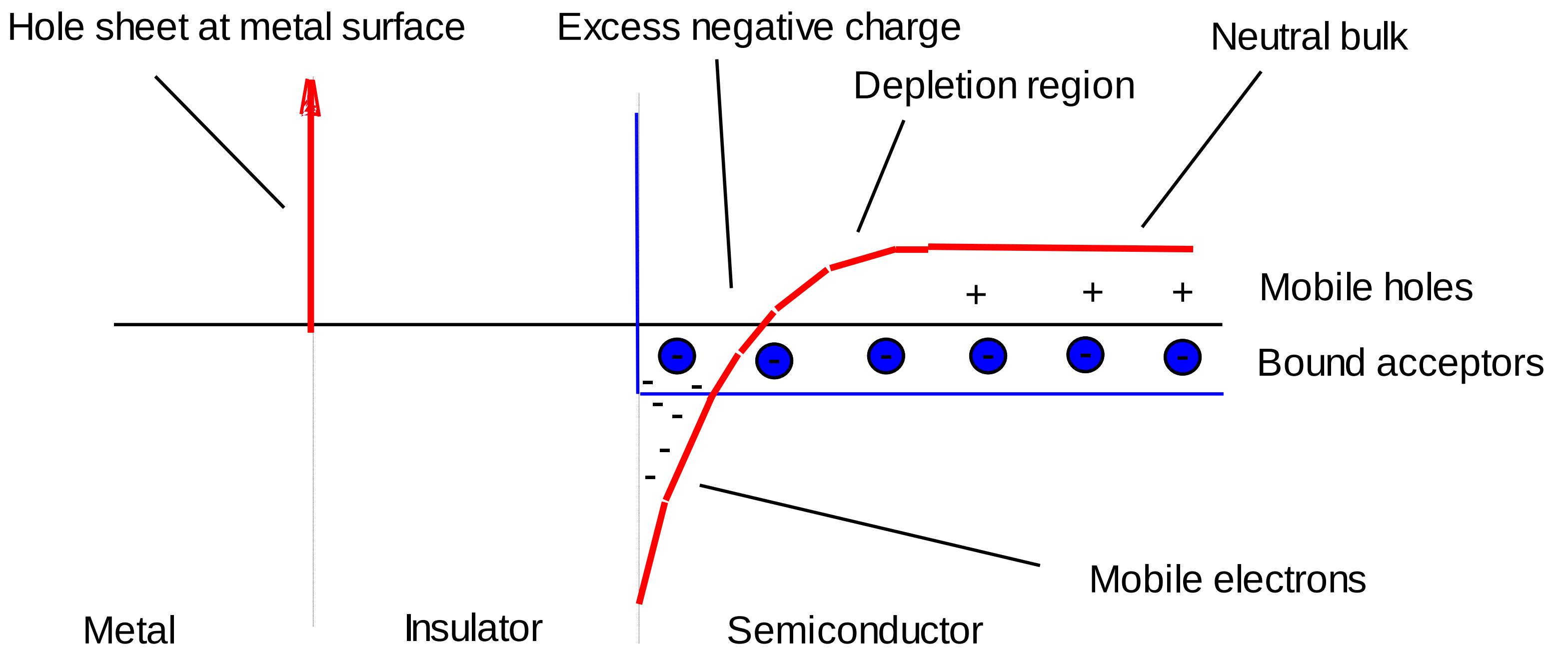


Electrons collect at the interface.

Due to the change in carrier type this is known as **inversion**.

Ideal MOS capacitor in inversion

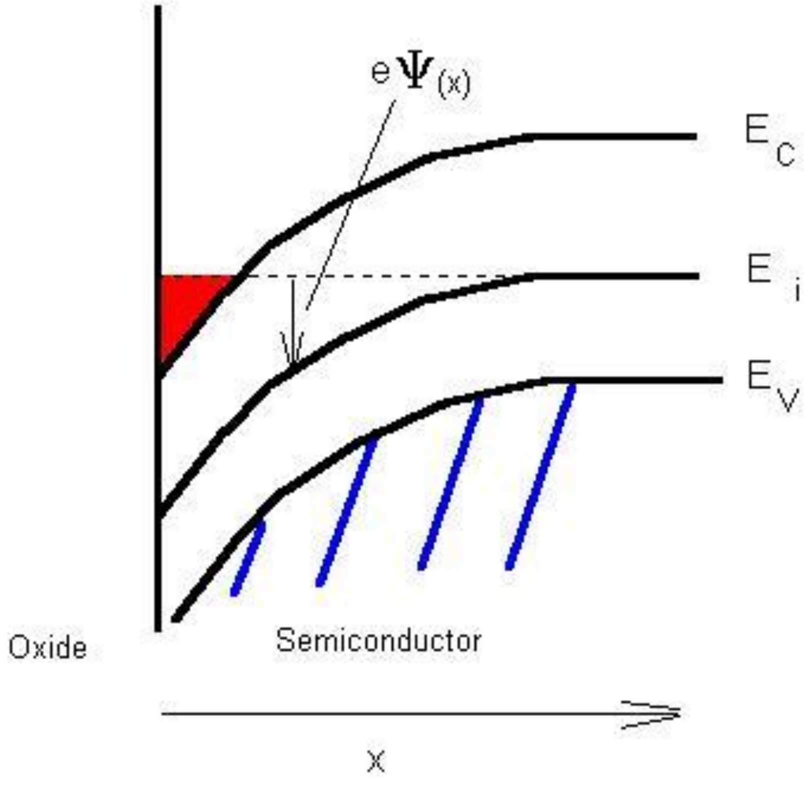
Charge distribution in the structure



Ideal MOS capacitor in inversion

Local potential in the semiconductor $\Psi_{(x)}$ with respect to the bulk material determines carrier concentrations.

Band diagram close to oxide-silicon interface.



$$n_p = n_i e^{\left[\frac{e\left(\Psi_{(x)} - \Psi_B\right)}{kT}\right]}$$

for electrons in the p-type regions.

$$p_p = n_i e^{\left[\frac{e\left(\Psi_B - \Psi_{(x)}\right)}{kT}\right]}$$

for holes in the p-type regions.

Ideal MOS capacitor in inversion

Strong inversion

Strong inversion defined to occur when induced carrier density $\mathbf{n_s}$ exceeds the bulk carrier density $\mathbf{N_a}$.

Threshold condition $$n_s = N_a$$

which occurs when $$\Psi_s = 2\Psi_B \approx \frac{2kT}{q}\ln\left(\frac{N_a}{n_i}\right)$$

Weak inversion condition

$$\Psi_B < \Psi_S < 2\Psi_B$$

Ideal MOS capacitor in inversion

MOSFETs (Metal Oxide Semiconductor Field Effect Transistors) are always used in Strong Inversion.

Inversion layer thickness

The charge density in the inversion layer increases with inversion thickness very rapidly so that the width of the inversion layer remains <~ 10nm under all conditions,

while the width of the depletion layer depends on the acceptor concentration.

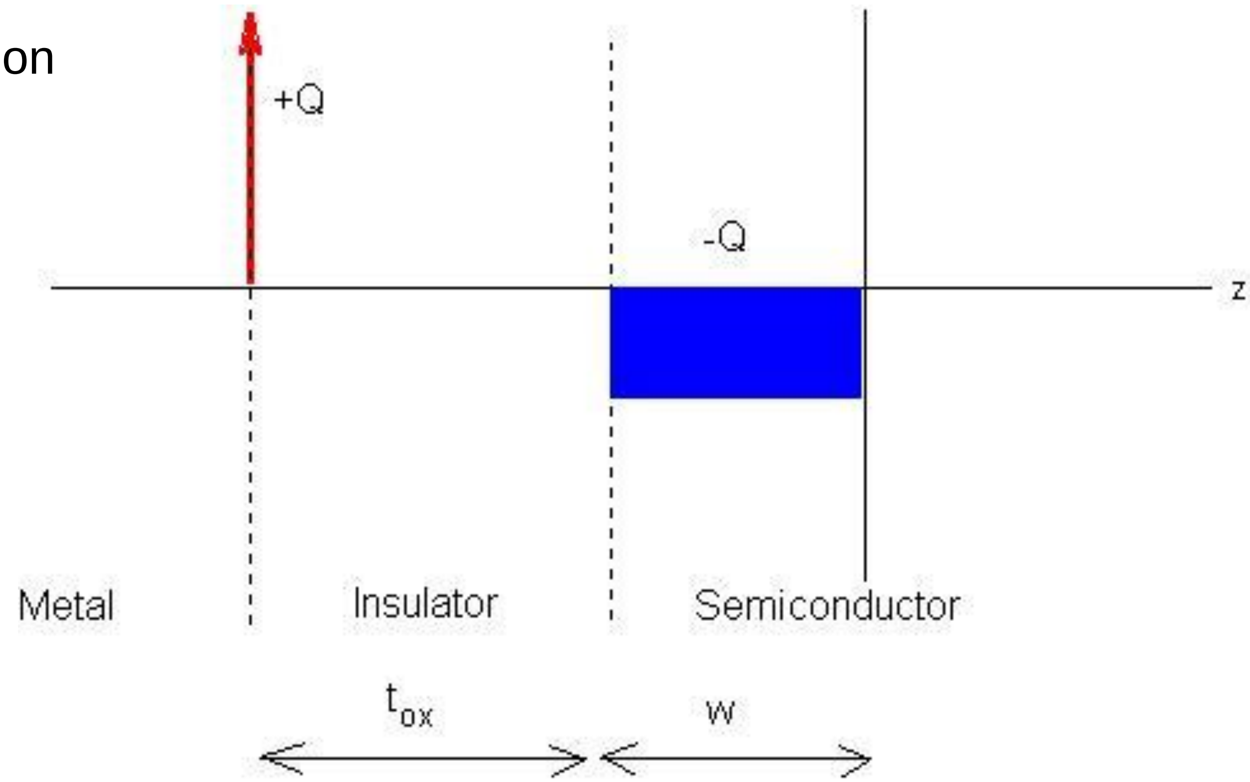
Ideal MOS capacitor-summary of behaviour

The surface potential characterises the nature of the charge at the oxide silicon interface.

$\Psi_s < 0$	accumulation of holes
$\Psi_s = 0$	flat band condition
$\Psi_B > \Psi_s > 0$	depletion of holes
$\Psi_s = \Psi_B$	mid-gap
$\Psi_s > \Psi_B$	inversion

Ideal MOS capacitance in depletion

Charge distribution

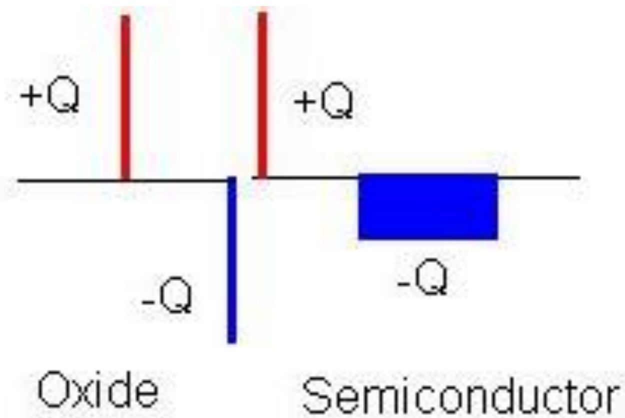


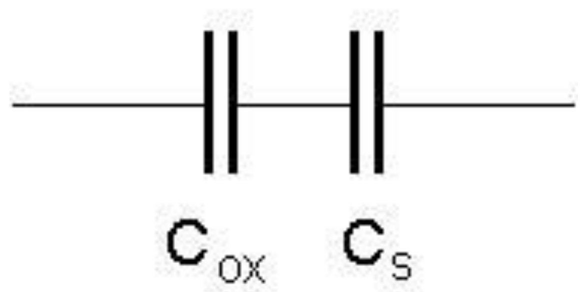
Magnitude of the charge in depletion layer

$$-Q = -ewN_a$$

Ideal MOS capacitance in depletion

Any applied voltage appears across the oxide and the depletion layer so that there are, in effect, two capacitors in series, i.e.





Ideal MOS capacitance in depletion

The oxide capacitance is given by $$C_{ox} = \frac{\varepsilon_o \varepsilon_{ox}}{t_{ox}}$$

The depletion layer capacitance is given by applying Poisson's equation

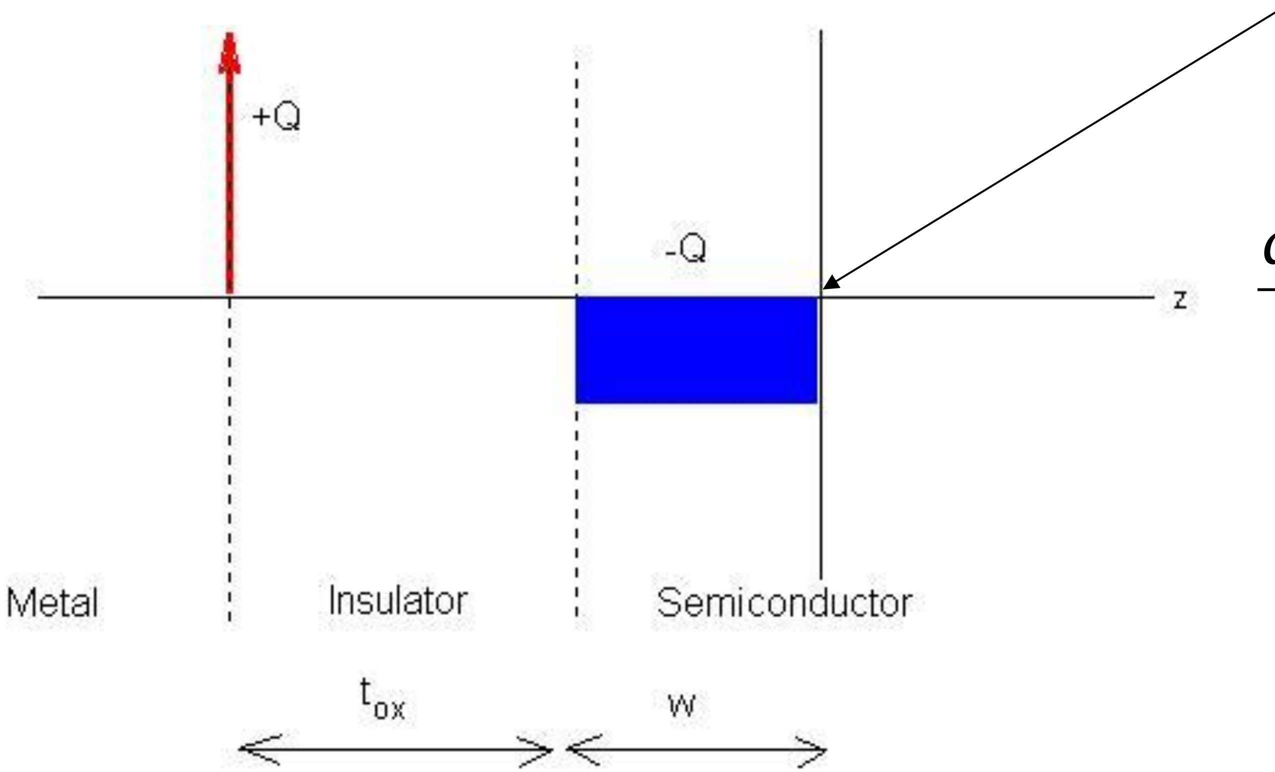
$$-\varepsilon_0 \varepsilon_s \frac{d^2 \Psi}{dz^2} = -eN_a$$

Ideal MOS capacitance in depletion

After integrating

$$\varepsilon_o \varepsilon_s \frac{d\Psi}{dz} = eN_a z$$

z-origin at edge of undepleted silicon, where



$$z = 0$$

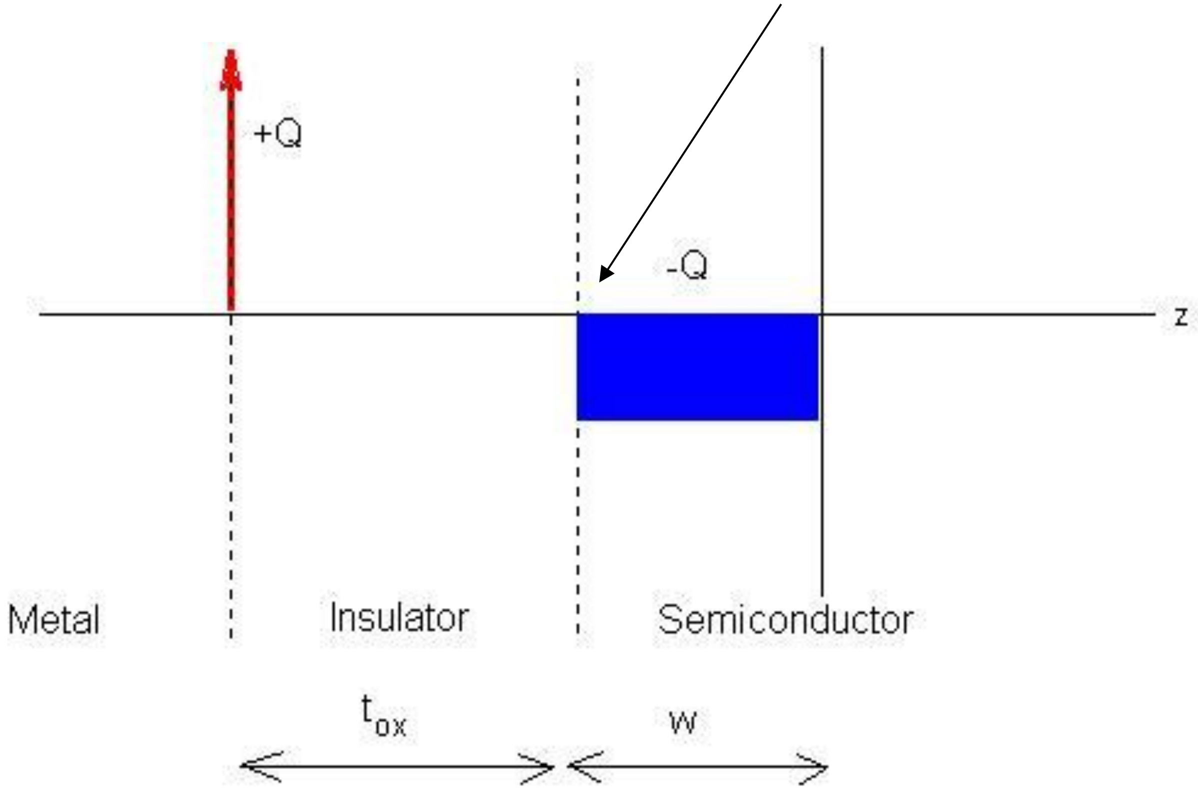
$$\Psi = 0$$

$$\frac{d\Psi}{dz} = 0$$

Ideal MOS capacitance in depletion

Gives the solution $$\varepsilon_o \varepsilon_s \Psi = \frac{1}{2} e N_a z^2$$

Now at the oxide-silicon interface $z = w$ and $\Psi = \Psi_s$



So that

$$\Psi_s = \frac{e N_a w^2}{2 \varepsilon_o \varepsilon_s}$$

This is the potential difference across the depletion layer

Ideal MOS capacitance in depletion

Charge **Q** given by

$$Q = ewN_a$$

So that

$$\Psi_s = \frac{Q^2}{2\varepsilon_o \varepsilon_s e N_a}$$

and

$$Q = \left(\Psi_s 2\varepsilon_o \varepsilon_s e N_a\right)^{\frac{1}{2}}$$

Ideal MOS capacitance in depletion

The small ac signal capacitance of the depletion layer is given by

$$C_s = \frac{dQ}{d\Psi_s} = \frac{1}{2}\left(\Psi_s\right)^{-\frac{1}{2}}\left(2\varepsilon_o\varepsilon_s eN_a\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(eN_a w^2\right)^{-\frac{1}{2}}\left(2\varepsilon_o\varepsilon_s\right)^{\frac{1}{2}}\left(2\varepsilon_o\varepsilon_s eN_a\right)^{\frac{1}{2}}$$

So that $C_s = \dfrac{\varepsilon_o\varepsilon_s}{w}$

Capacitance of parallel plate capacitor with gap equal to the depletion layer width and dielectric constant for silicon.

Ideal MOS capacitance in depletion

For the total capacitance **C** we must add these two capacitances in parallel, ie.

$$C = \frac{C_s C_{ox}}{\left(C_s + C_{ox}\right)}$$

and simplifying

$$\frac{C}{C_{ox}} = \frac{1}{\left(1 + \frac{C_{ox}}{C_s}\right)}$$

Ideal MOS capacitance in depletion

With an applied voltage **V** the potential difference is shared between the oxide and the depletion layer, so that

$$V = \Psi_s + \frac{Q}{C_{ox}} = \frac{eN_a w^2}{2\varepsilon_o \varepsilon_s} + \frac{eN_a w}{C_{ox}}$$

$$= \frac{eN_a \varepsilon_o \varepsilon_s}{\left(2C_s^2\right)} + \frac{eN_a \varepsilon_o \varepsilon_s}{C_s C_{ox}}$$

Ideal MOS capacitance in depletion

$$\frac{2C_{ox}^2 V}{(eN_a \varepsilon_o \varepsilon_s)} = \left(\frac{C_{ox}}{C_s}\right)^2 + 2\left(\frac{C_{ox}}{C_s}\right)$$ after rearrangement

and solving for **C_{ox} / C_s** we get $$\frac{C}{C_{ox}} = \left[1 + \frac{2V\varepsilon_{ox}^2 \varepsilon_o}{(eN_a \varepsilon_s t_{ox}^2)}\right]^{-\frac{1}{2}}$$

At zero bias $$\frac{C}{C_{ox}} = 1$$ ie. $$C = C_{ox}$$

This is the maximum capacitance.

The minimum capacitance occurs when the depletion layer has its maximum width **w_m**.

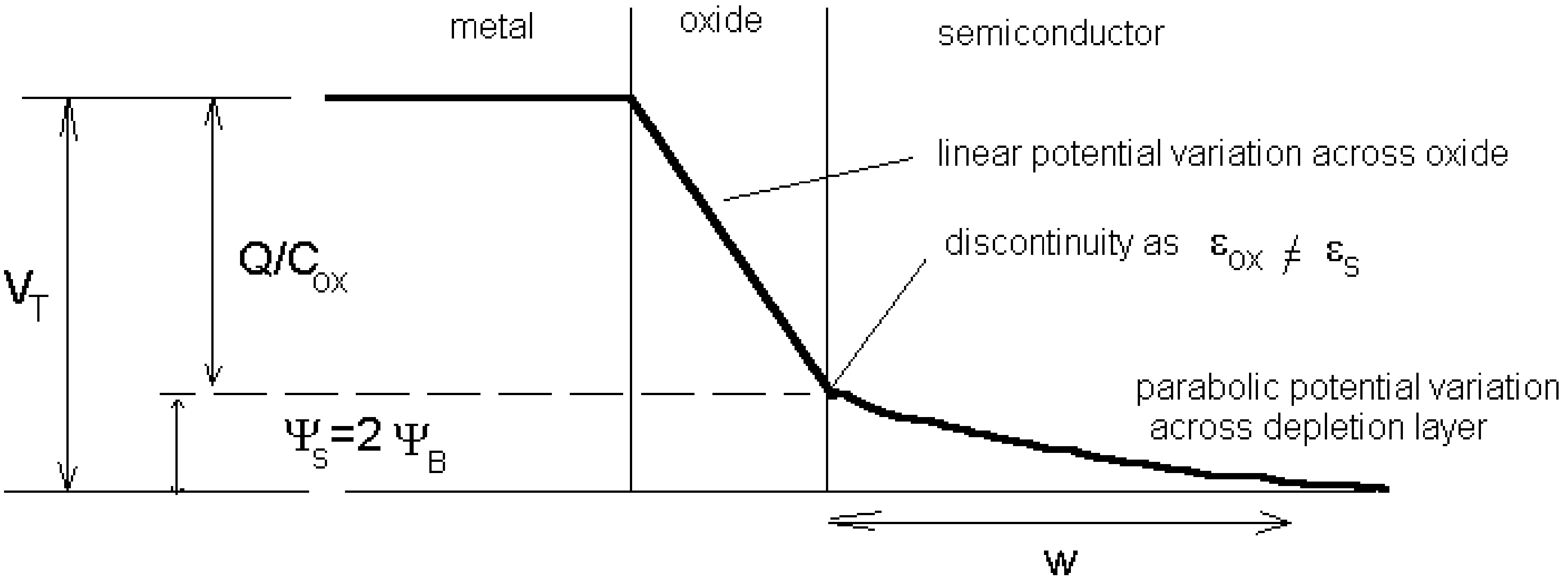
Ideal MOS capacitance in accumulation

The surface of the oxide is in electrical contact with the semiconductor bulk, so that

$$C = C_{ox}$$

Ideal MOS capacitance in inversion

Potential distribution at the onset of strong inversion



Threshold voltage V_T for strong inversion

The threshold voltage has two parts:

potential difference across the oxide

potential difference across the depletion region.

The threshold for strong inversion was **defined** as being when the surface potential is

$$\Psi_s = 2\Psi_B$$

So that

$$V_T = 2\Psi_B + \frac{Q}{C_{ox}}$$

but $Q = eN_a w_m$ and $w_m = \left(\frac{2\Psi_B 2\varepsilon_o \varepsilon_s}{eN_a} \right)^{\frac{1}{2}}$

Threshold voltage V_T for strong inversion

This gives $$V_T = 2\Psi_B + \frac{\left(2\Psi_B 2\varepsilon_o\varepsilon_s e N_a\right)^{\frac{1}{2}}}{C_{ox}}$$

Note that $$\Psi_B = \left(\frac{kT}{e}\right)\ln\left(\frac{N_a}{n_i}\right)$$

since $$p = N_a = n_i e^{\left(\frac{e\Psi_B}{kT}\right)}$$

Worked example

Calculate the maximum and minimum capacitance values for an ideal MOS structure with oxide (SiO_2) thickness of 0.1μm and substrate doping density of 1x10^{15}cm^{-3}. The maximum capacitance is given by that of the oxide alone ie

$$C_{ox} = \frac{\varepsilon_o \varepsilon_{ox}}{t_{ox}} = \frac{8.8\times 10^{-12}\times 3.9}{10^{-7}} = 3.43\times 10^{-4} Fm^{-2}$$

The minimum capacitance occurs when the depletion layer has its maximum width **w_m**. To find the maximum capacitance we need Ψ_B ie

$$\Psi_B = \left(\frac{kT}{e}\right)\ln\left(\frac{N_a}{n_i}\right) = \frac{1.4\times 10^{-23}\times 300}{1.6\times 10^{-19}}\ln\left(\frac{10^{15}}{10^{10}}\right)$$

$$= 0.026\times 11.5 = 0.3\,V$$

Worked example

The maximum depletion width is given by

$$w_m = \left(\frac{2\Psi_B 2\varepsilon_o \varepsilon_s}{eN_a} \right)^{\frac{1}{2}} = \left(\frac{2\times 0.3\times 2\times 8.8\times 10^{-12}\times 11.7}{1.6\times 10^{-19}\times 10^{21}} \right)^{\frac{1}{2}}$$

$$= 8.7\times 10^{-7}\, m$$

Worked example

The capacitance due to the depletion region is then

$$C_{sm} = \frac{\varepsilon_o \varepsilon_s}{w_m} = \frac{8.8 \times 10^{-12} \times 11.7}{8.7 \times 10^{-7}} = 11.8 \times 10^{-5}\, Fm^{-2}$$

The minimum capacitance is then given by

$$C_{\min} = \frac{C_{sm} C_{ox}}{\left(C_{sm} + C_{ox}\right)}$$

$$= \frac{11.8 \times 10^{-5} \times 3.43 \times 10^{-4}}{11.8 \times 10^{-5} + 3.43 \times 10^{-4}} = \frac{4.05 \times 10^{-8}}{4.61 \times 10^{-4}} = 8.8 \times 10^{-5}\, Fm^{-2}$$