4B6 EXAMPLE PAPER 1 SOLUTIONS

Q. 1

$$p_s = N_V \exp\left(\frac{E_V - E_F - q\psi_s}{kT}\right) = p_{po} \exp\left(\frac{-q\psi_s}{kT}\right)$$

From charge neutrality:

$$p_{po} = N_A^- \simeq N_A \quad \left(\begin{array}{c}\text{all acceptors}\\ \text{are ionised at}\\ T = 300°K\end{array}\right)$$

$$n_s = N_C \exp\left(\frac{E_F - E_C + q\psi_s}{kT}\right) = n_{no} \exp\left(\frac{q\psi_s}{kT}\right)$$

$$n_{no}\, p_{po} = n_i^2$$

For $q\psi_s = 0.25\,eV$

$p_s = 6.29 \times 10^{16}\, m^{-3}$ $n_s = 6.92 \times 10^{14}\, m^{-3}$

For $q\psi_s = 0.7\,eV$

$p_s = 1.72 \times 10^{9}\, m^{-3}$ $n_s = 2.53 \times 10^{22}\, m^{-3}$

b) One can answer directly noticing that:

for $\psi_s = 0.25$ $p_s, n_s \ll N_A$
so the MOS is in depletion

for $\psi_s = 0.7$ $p_s \ll N_A \ll n_s$
so the MOS is in inversion

Alternatively one can calculate $|E_F - E_i| = \psi_B$ and compare it with ψ_s. In the bulk ($\psi = 0$)

$$p = p_{po} = N_A = N_V \exp\left(-\frac{E_F - E_V}{kT}\right)$$

$$E_F - E_V = kT \ln \frac{N_V}{N_A} = 0.24\,eV$$

$$\psi_B = \frac{E_i - E_F}{q} = 0.56V - 0.24V = 0.32\ V$$

St. for $\psi_s < 0.32V$ - depletion

$0.32 < \psi_S \leq 0.64V$ - week inversion

$0.64 < \psi_S$ - strong inversion

Q.2

a) One has to solve Poisson's Equation

$$\frac{d^2\psi}{dx^2} = -\frac{\rho(x)}{\epsilon_s}$$

$\rho(x) = -qN_A$ independent of x

Integrating once:

$$\frac{d\psi}{dx} = q\frac{N_A}{\epsilon_s}x + C_1$$

For $x = a$ $\quad \frac{d\psi}{dx} = -\mathcal{E}$ (the electric field) $= 0$

depletion region length

because in the bulk the field is zero
therefore:

$$C_1 = -q\frac{N_A}{\epsilon_s}a$$

Integrating twice:

$$\psi(x) - \psi(a) = \frac{1}{2}q\frac{N_A}{\epsilon_s}(x-a)^2 - q\frac{N_A}{\epsilon_s}a(x-a)$$

Setting $\psi(a) = 0 \quad \psi(0) = \psi_s$

$$\psi_s = \frac{1}{2}q\frac{N_A}{\epsilon_s}a^2$$

$$a = \left(\frac{2\epsilon_s\psi_s}{qN_A}\right)^{1/2} = 5.7\ 10^{-7}\,m$$

b) Potential drop in the oxide

$Q_M = -Q_S$

$V_i = Q_M / C_i = (-Q_S / \epsilon_i) d$

$Q_S = -N_A a$

$V_i = \frac{N_A a d}{\epsilon_i} = 0.08\ V$

c) gate voltage required to give $\psi_S = 0.25$

$V_G = V_i + \psi_S = 0.33\ V$

Q. 3

a) We consider a MOS capacitor on a p-type semiconductor. When a positive voltage step, sufficient to produce strong inversion, is applied to the gate, minority carriers are thermally generated, resulting in the formation of the inversion layer. The generation process, however, requires a finite time, of the order of a millisecond. Therefore, before thermal equilibrium is reached there are neither holes nor electrons in a region next to the oxide/semiconductor interface, since the former are repelled due to the positive gate voltage and the latter have not yet been generated. Such region is therefore depleted of carriers and the regime is referred to as 'deep depletion' because the depletion region is larger than in thermal equilibrium.

b) $V_G = V_i + \psi_S$, $V_i = -\frac{Q_S}{\epsilon} d$

$Q_n = 0 \quad Q_S = Q_B$

$$Q_B = -qN_A a$$

a = depletion region length

a (see Q. 2) is given by:

$$a = \left(\frac{2\epsilon_s \psi_s}{qN_A}\right)^{1/2}$$

$$Q_B = -(2qN_A\epsilon_s\psi_s)^{1/2}$$

$$V_i = \frac{d}{\epsilon_i}(2qN_A\epsilon_s)^{1/2}\psi_s^{1/2} = B\psi_s^{1/2}$$

$$V_G = B\psi_s^{1/2} + \psi_s$$

$$\psi_s^{1/2} = -\frac{B}{2} + \sqrt{\frac{B^2}{4} + V_G}$$

$$\psi_s = 3.94\ V$$

$$Q_B = -3.64\times10^{-8}\ C\ cm^{-2}$$

The maximum field in the semiconduc[tor] is at the interface.

$$F_s = \frac{Q_B}{\epsilon_s} = -3.46\times10^4\ V\ cm^{-1}$$

the field in the oxide is constant:

$$F_{ox} = \frac{F_s\epsilon_s}{\epsilon_i} = \frac{Q_B}{\epsilon_i} = 1.05\times10^5\ V[\ldots]$$

Note that $F_{ox} = \dfrac{V_G-\psi_s}{d}$ gives the same res[ult]

Q. 4

The equation in this question is obtained from equations 16, 13, and 12 of Section 1 of the notes, by assuming:

$$\frac{n_{po}}{p_{po}} e^{\beta \psi_s} \gg \beta \psi_s \gg 1$$

The equation however is not needed here. In fact:

$$V_G = V_i + \psi_s = -\frac{Q_s}{\epsilon_i} d + \psi_s \simeq -\frac{Q_s}{\epsilon_i} d$$

The maximum field in the semiconductor is at the interface:

$$F_s = \frac{Q_s}{\epsilon_s} = -\frac{\epsilon_i}{\epsilon_s} \frac{V_G}{d} = -1.64 \times 10^5$$

The field in the oxide is constant and given by:

$$F_{ox} = \frac{\epsilon_s F_s}{\epsilon_i} = \frac{V_G}{d} = -5 \times 10^5$$

Note that rigorously

$$F_{ox} = \frac{V_G - \psi_s}{d},$$

but in this question we have assumed $-\frac{Q_s}{\epsilon_i} d \gg \psi_s$.

Q.5

Since ψ_s is the same as in Q.2, the depletion length a is the same. (what changes in the presence of a fixed charge Q_f is the gate voltage needed to produce a given ψ_s).

Now:

$$Q_M = -(-N_A a + Q_f)$$

$$V_i = (N_A a - Q_f)\frac{d}{\epsilon_i}$$

For $Q_f = +10^{-4}\,C\,m^{-2}$ $\quad V_i = -0.0071\ V$

$V_G = V_i + \psi_s = 0.243\ V$

For $Q_f = -10^{-4}\,C\,m^{-2}$ $\quad V_i = 0.167\ V$

$V_G = 0.417\ V$

Q.6 We consider first the case $Q_f > 0$

$$V_G = -\frac{d}{\epsilon_i}(Q_s + Q_f) + \psi_s$$

$$\left(V_G + \frac{d}{\epsilon_i} Q_f\right) = -\frac{d}{\epsilon_i} Q_s + \psi_s \qquad (1)$$

Since $|Q_s|$ is now higher than for the case of Q.4, where $Q_f = 0$, we can again neglect ψ_s. So we have:

$$Q_s \simeq -\frac{\epsilon_i}{d}\left(V_G + \frac{d}{\epsilon_i} Q_f\right) = \epsilon_s F_s$$

Hence:

$$F_s = -3.16 \times 10^5$$

$$F_{ox} = -\frac{Q_s + Q_f}{\epsilon_i} \simeq -\frac{V_G}{d} = -5\times10^5$$

The case $Q_f < 0$ is more complicated and you can omit it if you wish.

Looking at equation (1) above, one can s that $|Q_s|$ is now lower than for the case $Q_f = 0$. Hence we cannot assume inversion. To determine in what regime the MOS is operating we calculate the threshold voltage (voltage corresponding to a surface potential equal to $2\psi_B$)

$$\psi_B = \frac{E_i - E_F}{q} = \frac{kT}{q} \log\left(\frac{N_A}{n_i}\right) = 0.[illegible]$$

$$V_T = 2\psi_B - \frac{Q_f + Q_B}{\epsilon_i} d$$

From Q. 3

$$Q_B = -\left(2q N_A \epsilon_s \psi_s\right)^{1/2}$$

$$= -\left(2q N_A \epsilon_s 2\psi_B\right)^{1/2} = -1.41 \times 10^{-8} \, C\,cm^{-2}$$

$$V_T = 5.64 \, eV$$

Hence for $V_G = 5V$, the MOS is operat below threshold for strong inversion and

$$Q_s \simeq Q_B$$

Therefore:

$$V_G = \psi_s - \frac{Q_f + Q_B}{\epsilon_i} d$$

$$\left(V_G + \frac{Q_f}{\epsilon_i} d\right) = \psi_s + \frac{d}{\epsilon_i}(2qN_A\epsilon_s)^{1/2}\psi_s^{1/2} =$$

$$= \psi_s + B\psi_s^{1/2}$$

$$\psi_s^{1/2} = -\frac{B}{2} + \sqrt{\frac{B^2}{4} + \left(V_G + \frac{Q_f}{\epsilon_i} d\right)}$$

$$\psi_s = 0.156\,V$$

$$Q_B = -(2qN_A\epsilon_s\psi_s)^{1/2} = -7.23 \times 10^{-9}$$

$$F_s = \frac{Q_B}{\epsilon_s} = -6.86 \times 10^{3}$$

$$F_{ox} = -\frac{V_G - \psi_s}{d} = -4.845 \times 10^{5}$$

Q. 7

We consider a MOSFET with a negligible but finite source-drain voltage V_{DS}. The fact that V_{DS} is very small, insures the the channel potential V_c is nearly constant. Therefore, the surface potential, for a given gate voltage V_G is also constant.

The threshold voltage V_T is defined as the gate-source voltage for which $\psi_s = 2\psi_B$ (where $\psi_B = \frac{E_i - E_F}{q}$)

For $\psi_s = 2\psi_B$ we have:

$$V_{GC} = V_G - V_C \simeq V_{GS} = 2\psi_B + V_i$$

$$V_i = (N_A a - Q_f)\frac{d}{\epsilon_0 \epsilon_i} \quad \text{(see Q. 3)}$$

$$a = \left[\frac{2\epsilon_s(2\psi_B)}{q N_A}\right]^{1/2} = 9.2\times10^{-7}\,\text{m} \quad \text{(see Q. 2 a)}$$

$\psi_B = 0.32$ V (see Q. 1). Substituting the other numerical values one obtains:

$V_T = 0.77$V for $Q_f = 0$, $V_T = 0.68$V for $Q_f = 10^{-4}$ C m^{-2}

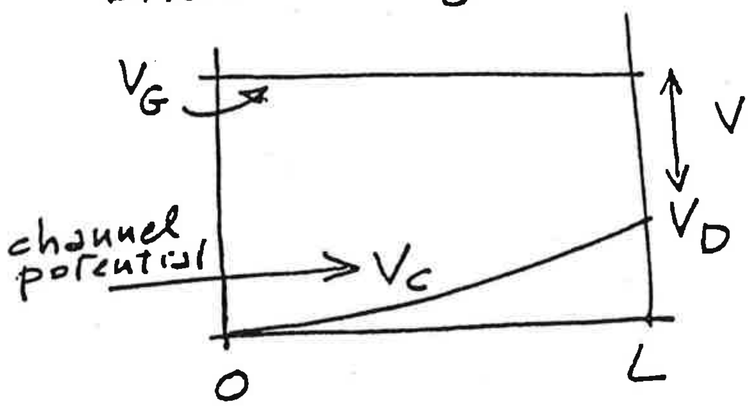
Q. 8

When a voltage is applied to the gate and the drain, the Fermi energy $E_F(x,y)$ becomes a function of both x (the coordinate in the direction perpendicular to the interface and y (the direction parallel to the channel). We take as usual $x=0$ at the oxide/semiconductor interface and $y=0$ at the source end, $y=L$ at the drain end. The 'channel potential' V_c is defined.

$$V_c(y) = \frac{E_F(0,0) - E_F(0,y)}{q}$$

$$V_c(L) = V_{DS} = \frac{E_F(0,0) - E_F(0,L)}{q}$$

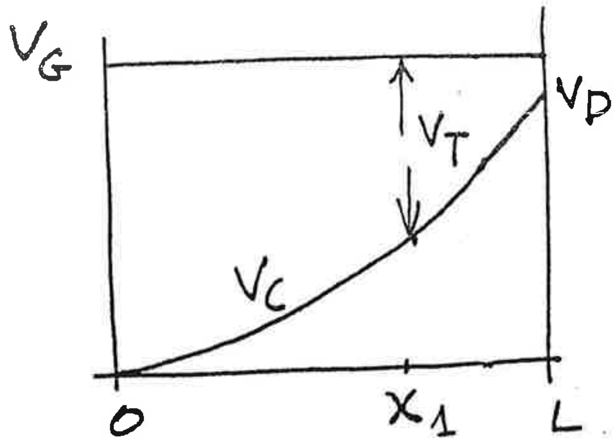
'Linear' regime



$$V_G - V_D > V_T$$

This means that there is inversion at every point in the channel.

'Saturation' regime

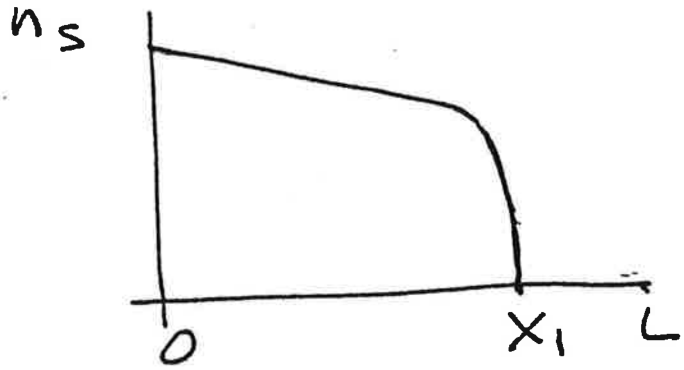


$V_G - V_D < V_T$ so that:

nversion for $0 < x < x_1$ $\quad V_G - V_C \geq V_T$

o inversion for $x_1 < x < L$ $\quad V_G - V_C < V_T$

Electron concentration near the surface
(a NMOS is assumed)



x_1, corresponding to the condition $V_C = V_{Dsat}$ is the pinch-off point.

Q. 9

$$I_D = W \frac{dV_c}{dy} G(y).$$

$$= W \frac{dV_c}{dy} \mu_n \frac{\varepsilon_i}{d} (V_{GS} - V_c - V_T)$$

Integrating the above equation by separation of variables, one has, since I_D is constant:

$$I_D \int_0^y dy' = W \mu_n \frac{\varepsilon_i}{d} \int_0^{V_c} (V_{GS} - V_c' - V_T)\, dV_c'$$

$$I_D y = W \mu_n \frac{\varepsilon_i}{d} \left[(V_{GS} - V_T) V_c - \frac{1}{2} V_c^2 \right] \quad (1)$$

For $y = L$:

$$I_D L = W \mu_n \frac{\varepsilon_i}{d} \left[(V_{GS} - V_T) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \quad (2)$$

Dividing (1) by (2):

$$\frac{y}{L} = \frac{(V_{GS} - V_T) V_c - \frac{1}{2} V_c^2}{(V_{GS} - V_T) V_{DS} - \frac{1}{2} V_{DS}^2}$$

solving for V_c and discarding the solution $V_c > V_{DS}$ one has:

$$V_c = 2.08\ \mathrm{V}$$